

Rate and Power Allocation for Discrete-Rate Link Adaptation

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Abstract—Link adaptation, in particular adaptive coded modulation (ACM), is a promising tool for bandwidth-efficient transmission in a fading environment. The main motivation behind employing ACM schemes is to improve the spectral efficiency of wireless communication systems. In this paper, using a finite number of capacity achieving component codes, we propose new transmission schemes employing constant power transmission, as well as discrete and continuous power adaptation, for slowly varying flat-fading channels.

We show that the proposed transmission schemes can achieve throughputs close to the Shannon limits of flat-fading channels using only a small number of codes. Specifically, using a fully discrete scheme with just four codes, each associated with four power levels, we achieve a spectral efficiency within 1 dB of the continuous-rate continuous-power Shannon capacity. Furthermore, when restricted to a fixed number of codes, the introduction of power adaptation has significant gains with respect to ASE and probability of no transmission compared to a constant power scheme.

I. INTRODUCTION

In wireless communications bandwidth is a scarce resource. By employing link adaptation, in particular adaptive coded modulation (ACM), we can achieve bandwidth-efficient transmission schemes. Today, adaptive schemes are already being implemented in wireless systems such as Digital Video Broadcasting - Satellite Version 2 (DVB-S2) [1]. A generic ACM system [2]–[10] is illustrated in Fig. 1. Such a system adapts to the channel variations by utilizing a set of component channel codes and modulation constellations with different spectral efficiencies (SEs).

We consider a wireless channel with additive white Gaussian noise (AWGN) and fading. Under the assumption of slow, frequency-flat fading, a block-fading model can be used to approximate the wireless fading channel by an AWGN channel

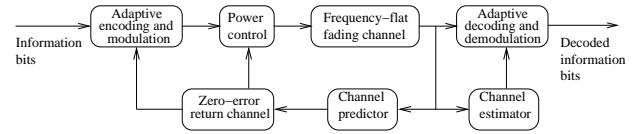


Fig. 1. Adaptive coded modulation system.

within the length of a codeword [11], [12]. Hence, the system may use codes which typically guarantee a certain spectral efficiency within a range of signal-to-noise ratios (SNRs) on an AWGN channel. At specific time instants, a prediction of the instantaneous SNR is utilized to decide the highest-SE code that can be used. The system thus compensates for periods with low SNR by transmitting at a low SE, while transmitting at a high SE when the SNR is favorable. In this way, a significant overall gain in *average spectral efficiency* (ASE)—measured in information bits/s/Hz—can be achieved compared to fixed rate transmission systems.

In the current literature we can identify two main approaches to the design of adaptive systems with a finite number of transmission rates [2], [13]–[18]. One key point is the starting point for the design. In [16]–[18] the problem can be stated as follows: Given that the system quantizes any channel state to one of L levels, what is the maximum spectral efficiency that can be obtained using discrete rate signalling? On the other hand, in [2], [13]–[15] the question is: Given that the system can utilize N transmission rates, what is the maximum spectral efficiency? Another key difference is that in [2], [13]–[15] the system is designed to maximize the average spectral efficiency according to a *zero information outage* principle, such that at poor channel conditions transmission is disabled, and data buffered. However, in [16]–[18], data are allowed to be transmitted at all time instants, and an *information outage* occurs when the mutual information offered by the channel is lower than the transmitted rate. While seemingly similar, these approaches actually leads to different designs as will be demonstrated. Though allowing for a non-zero outage can offer more flexibility in the design, it also comes with the drawbacks of losing data and spilling system resources (e.g., power). Furthermore, in [16]–[18] the important issues of how often data are lost due to an information outage, and how to deal with it are not discussed, e.g., many applications would then require the communication system to be equipped with a retransmission capability. These differences render a fair comparison between the approaches difficult; however we provide a numerical example later to illustrate the key points

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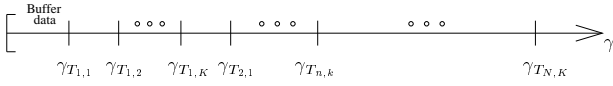


Fig. 2. The pre-adaptation SNR range is partitioned into regions where $\gamma_{T_{n,k}}$ are the switching thresholds.

above.

In [16]–[18] adaptive transmission with a finite number of capacity-achieving codes, and a single power level per code are considered. However, from previous work by Chung and Goldsmith [6] we know that the spectral efficiency of such a restricted adaptive system increases if more degrees of freedom are allowed. In particular, for a finite number of transmission rates, power control is expected to have a significant positive impact on the system performance, and hence in this paper we propose and analyze more flexible power control schemes, for which the single power level per code scheme of [16]–[18] can be seen as a special case.

In this paper we focus on data communications which, as emphasized in [19], cannot “tolerate any loss”. For such applications it thus seems more reasonable to follow the zero information outage design philosophy of [2], [13]–[15]. This choice is also supported by the work done in the design of adaptive coding and modulation for real-life systems, e.g., in DVB-S2 [1]. Based on this philosophy we derive transmission schemes that are optimal with regard to maximal ASE for a given fading distribution. By the assuming codes to be operating at AWGN channel capacity, we formulate constrained ASE maximization problems and proceed to find the optimal switching thresholds and power control schemes as their solutions. Considering both constant power transmission as well as discrete and continuous power adaptation, we show that the introduction of power adaptation provides substantial average spectral efficiency and probability of no transmission gains when the number of rates is finite. Specifically, spectral efficiencies within 1 dB of the continuous-rate continuous-power Shannon capacity are obtained using a completely discrete transmission scheme with only four codes and four power levels per code.

The remainder of the present paper is organized as follows. We introduce the wireless model under investigation and describe the problem under study in Section II. Optimal transmission schemes for link adaptation are derived and analyzed in Section III. Numerical examples and plots are presented in Section IV. Finally, conclusions and discussions are given in Section V.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Model

We consider the single-link wireless system depicted in Fig. 1. The discrete-time channel is a stationary fading channel with time-varying gain. The fading is assumed to be slowly varying and frequency-flat. Assuming, as in [2], [20], that the transmitter receives perfect channel predictions we can adapt the transmit power instantaneously at time i according to a power adaptation scheme $S(\cdot)$. Then, denote the instantaneous *pre-adaptation* received signal-to-noise ratio (SNR) by $\gamma[i]$,

and the average *pre-adaptation* received SNR by $\bar{\gamma}$. These are the SNRs that would be experienced using signal constellations of average power \bar{S} without power control [4]. Adapting the transmit power based on the channel state $\gamma[i]$, the received SNR after power control, termed *post-adaptation* SNR, at time i is then given by $\gamma[i]S(\gamma[i])/\bar{S}$. By virtue of the stationarity assumption, the distribution of $\gamma[i]$ is independent of i , and is denoted by $f_\gamma(\gamma)$. To simplify the notation we omit the time reference i from now on.

Following [2], [21], we partition the range of γ into $NK+1$ *pre-adaptation* SNR regions, which are defined by the switching thresholds $\{\gamma_{T_{n,k}}\}_{n,k=1,1}^{N,K}$, as illustrated in Fig. 2. Code n , with spectral efficiency R_n , is selected whenever γ is in the interval $[\gamma_{T_{n,1}}, \gamma_{T_{n+1,1}})$, $n = 1, \dots, N$. Within this interval the transmission rate is constant, but the system can adapt the transmitted power, subject to an average power constraint of \bar{S} , to the channel conditions in order to maximize the average spectral efficiency. If the *pre-adaptation* SNR is below $\gamma_{T_{1,1}}$, data are buffered. For convenience, we let $\gamma_{T_{0,1}} = 0$ and $\gamma_{T_{N+1,1}} = \infty$.

B. Problem Formulation

The capacity of an AWGN channel is well known to be $C(\gamma) = \log_2(1 + \frac{S(\gamma)}{\bar{S}}\gamma)$ information bits/s/Hz, where $\frac{S(\gamma)}{\bar{S}}\gamma$ is the received SNR. This means that there exist codes that can transmit with arbitrarily small error rate at all spectral efficiencies up to $C(\gamma)$ bits/s/Hz, provided that the received SNR is (at least) $\frac{S(\gamma)}{\bar{S}}\gamma$. Our goal is now to find an optimal set of capacity-achieving transmission rates, switching levels, and power adaptation schemes in order to maximize the average spectral efficiency for a given fading distribution.

Using the results of [16], an information outage can only occur for a set of channel states within the first interval, which in our setup corresponds to that data should only be buffered for channel states in the first interval. Whereas in the other SNR regions, the assigned rate supports the worst channel state of that region. The average spectral efficiency of the system (in information bit per channel use) can then be written as

$$\bar{R} = \sum_{n=1}^N R_n P_n, \quad (1)$$

where $P_n = \int_{\gamma_{T_{n,1}}}^{\gamma_{T_{n+1,1}}} f_\gamma(\gamma) d\gamma$, which is the probability that code n is used.

III. OPTIMAL DESIGN FOR MAXIMUM AVERAGE SPECTRAL EFFICIENCY

Based on the above setup, we now proceed to design spectral efficiency maximizing schemes. Recall that the *pre-adaptation* SNR range is divided into regions lower bounded by $\gamma_{T_{n,1}}$, for $n = 0, 1, \dots, N$. Thus, we let $R_n = C_n$, where $C_n = \log_2(1 + \frac{S(\gamma_{T_{n,1}})}{\bar{S}}\gamma_{T_{n,1}})$ is shown below to be the highest spectral efficiency that can be supported within the range $[\gamma_{T_{n,1}}, \gamma_{T_{n+1,1}})$ for $1 \leq n \leq N$, after transmit power adaptation. Note that the fading is nonergodic within each codeword, so that the results of [22, Section IV] do not apply.

An upper bound on the ASE of the ACM scheme—for a given set of codes/switching levels—is therefore the *maximum ASE for ACM* (MASA), defined as

$$\begin{aligned} \text{MASA} &= \sum_{n=1}^N C_n P_n \\ &= \sum_{n=1}^N \log_2 \left(1 + \frac{S(\gamma_{T_{n,1}})}{\bar{S}} \gamma_{T_{n,1}} \right) \int_{\gamma_{T_{n,1}}}^{\gamma_{T_{n+1,1}}} f_\gamma(\gamma) d\gamma, \end{aligned} \quad (2)$$

subject to the average power constraint,

$$\sum_{n=0}^N \int_{\gamma_{T_{n,1}}}^{\gamma_{T_{n+1,1}}} S(\gamma) f_\gamma(\gamma) d\gamma \leq \bar{S}, \quad (3)$$

where \bar{S} denotes the average transmit power. (2) is basically a discrete-sum approximation of the integral expressing the Shannon capacity in [20, Eq. (4)]. If arbitrarily long codewords can be used, the bound can be approached from below with arbitrary precision for an arbitrarily low error rate. Using N distinct codes we analyze the MASA for constant, discrete, and continuous transmit power adaptation schemes, deriving the optimal rate and power adaptation for maximizing the average spectral efficiency. We shall assume that the fading is so slow that capacity-achieving codes for AWGN channels can be employed, giving tight bounds on the MASA [23], [24].

A. Continuous-Power Transmission Scheme

In an ideal adaptive power control scheme, the transmitted power can be varied to entirely track the channel variations. Then, for the N regions where we transmit, we show that the optimal continuous power adaptation scheme is *piecewise channel inversion*¹ to keep the received SNR constant within each region, much like the bit error rate is kept constant in optimal adaptation for constellation restrictions in [2]. For each rate region we use a capacity-achieving code which ensures an arbitrarily low probability of error for any AWGN channel with a received SNR greater than or equal to $\frac{S(\gamma_{T_{n,1}})}{\bar{S}} \gamma_{T_{n,1}} \triangleq \kappa_n$. The optimality of this strategy is formally proven below.

Lemma 1: For the $N + 1$ SNR regions the optimal continuous power control scheme is of the form

$$\frac{S(\gamma)}{\bar{S}} = \begin{cases} \frac{\kappa_n}{\gamma}, & \text{if } \gamma_{T_{n,1}} \leq \gamma < \gamma_{T_{n+1,1}}, 1 \leq n \leq N, \\ 0, & \text{if } \gamma < \gamma_{T_{1,1}}, \end{cases} \quad (4)$$

where $\{\kappa_n, \gamma_{T_{n,1}}\}_{n=1}^N$ are parameters to be optimized.

Proof: Assume for the purpose of contradiction that the power scheme given in (4) is not optimal, i.e., it uses too much power for a given rate. Then, by assumption, there exists at least one point in the set

$$\bigcup_{n=1}^N \{\gamma : \gamma_{T_{n,1}} \leq \gamma < \gamma_{T_{n+1,1}}\} \quad (5)$$

¹The results of this section were in part presented in [25]. Similar results on continuous power adaptation, when allowing for information outage (loss of data), were also later independently reported in [26].

where it is possible to use less power; denote this point by γ' . Fix any $\epsilon > 0$ and let $\frac{S(\gamma')}{\bar{S}} = \frac{\kappa_n}{\gamma'} - \epsilon$. This yields a received SNR of $\kappa_n - \epsilon\gamma' < \kappa_n$, but is less than the minimum required SNR for a rate of $\log_2(1 + \kappa_n)$. Hence, it does not exist any point where the proposed power scheme can be improved, and the assumption is contradicted. ■

Using (4) the received SNR, after power adaptation, is then for $n = 1, 2, \dots, N$ given as:

$$\frac{S(\gamma)}{\bar{S}} \gamma = \begin{cases} \kappa_n, & \text{if } \gamma_{T_{n,1}} \leq \gamma < \gamma_{T_{n+1,1}}, \\ 0, & \text{if } \gamma < \gamma_{T_{1,1}}, \end{cases} \quad (6)$$

i.e., we have a constant received SNR of κ_n within each region, supporting a maximum spectral efficiency of $\log_2(1 + \frac{S(\gamma_{T_{n,1}})}{\bar{S}} \gamma_{T_{n,1}}) = \log_2(1 + \kappa_n)$.

Introducing the power adaptation scheme (4) in (2), (3), and changing the average power inequality to an equality for maximization, we arrive at a scheme we denote $\text{MASA}_{N \times \infty}$, posing the following optimization problem²:

$$\text{maximize } \text{MASA}_{N \times \infty} = \sum_{n=1}^N \log_2(1 + \kappa_n) w_n \quad (7a)$$

$$\text{s.t. } \sum_{n=1}^N \kappa_n c_n = 1, \quad (7b)$$

where we have introduced the notation $w_n = \int_{\gamma_{T_{n,1}}}^{\gamma_{T_{n+1,1}}} f_\gamma(\gamma) d\gamma$, $c_n = \int_{\gamma_{T_{n,1}}}^{\gamma_{T_{n+1,1}}} \frac{1}{\gamma} f_\gamma(\gamma) d\gamma$. Note that for $N = 1$, (7) reduces to the truncated channel inversion Shannon capacity scheme given in [20, Eq. 12]. Inspecting (7), we see that for a given set of $\{\gamma_{T_{n,1}}\}$, the problem is a standard convex optimization problem in $\{\kappa_n\}$ with a waterfilling solution given as [27]

$$\kappa_n = \frac{w_n}{\lambda c_n} - 1, \quad n = 1, \dots, N, \quad (8)$$

where λ is a Lagrange multiplier to satisfy the average power constraint, and can from (7b) be expressed as a function of the switching thresholds as

$$\lambda = \frac{1 - F_\gamma(\gamma_{T_{1,1}})}{1 + \sum_{n=1}^N c_n}, \quad (9)$$

where $F_\gamma(\cdot)$ denotes the cumulative distribution function (cdf) of γ . Finally, the optimal values of $\{\gamma_{T_{n,1}}\}$, are found by equating the gradient of $\text{MASA}_{N \times \infty}$ to zero,

$$\nabla \text{MASA}_{N \times \infty} = \nabla \sum_{n=1}^N \log_2 \left(\frac{w_n}{\lambda c_n} \right) = \mathbf{0}. \quad (10)$$

The solution to (10) is found through numerical procedures, reducing the $2N$ -dimensional constrained optimization problem to solving N equations in N unknowns. Numerical results for the resulting adaptive power policy and the corresponding spectral efficiencies are presented in Section IV.

²Strictly speaking, we should add the constraints $0 \leq \gamma_{T_{1,1}} \leq \dots \leq \gamma_{T_{N,1}}$, and $\kappa_n \geq 0, \forall n$. However, we instead verify that the solutions we find satisfy these constraints.

B. Discrete-Power Transmission Scheme

For practical scenarios the resolution of power control will be limited, e.g., for the Universal Mobile Telecommunications System (UMTS) power control step sizes on the order of 1 dB are proposed [28]. We thus extend the MASA analysis by considering discrete power adaptation. Specifically, we introduce the $MASA_{N \times K}$ scheme where we allow for $K \geq 1$ power regions *within* each of the N rate regions. For each rate region we again use a capacity-achieving code for any AWGN channel with a received SNR greater than or equal to $\frac{S(\gamma_{T_{n,1}})}{S} \gamma_{T_{n,1}} = \kappa_n$. The optimal discrete power adaptation is discretized piecewise channel inversion, closely related to the discrete power scheme in [14].

Lemma 2: The optimal discrete-power adaptation scheme is of the form

$$\frac{S(\gamma)}{S} = \begin{cases} \frac{\kappa_n}{\gamma_{T_{n,k}}}, & \text{if } \gamma_{T_{n,k}} \leq \gamma < \gamma_{T_{n,k+1}}, \\ 1 \leq n \leq N, 1 \leq k \leq K \\ 0, & \text{if } \gamma < \gamma_{T_{1,1}}, \end{cases} \quad (11)$$

where $\gamma_{T_{n,K+1}} \triangleq \gamma_{T_{n+1,1}}$, $\{\kappa_n\}_{n=1}^N$ and $\{\gamma_{T_{n,k}}\}_{n=1, k=1,1}^{N,K}$ are the parameters to be optimized.

Proof: To ensure reliable transmission in each rate region $1 \leq n \leq N$, we require $\frac{S(\gamma)}{S} \gamma \geq \kappa_n$, assuming $\gamma \in [\gamma_{T_{n,1}}, \gamma_{T_{n+1,1}})$. Thus, following the proof of Lemma 1, since the rate is restricted to be constant in each region, it is obviously optimal from a capacity maximization perspective to reduce the transmitted power, when the channel conditions are more favorable. (11) is then obtained by reducing the power in a stepwise manner ($K-1$ steps), and at each step obtaining a received SNR of κ_n , i.e., $\frac{S(\gamma_{T_{n,k}})}{S} \gamma_{T_{n,k}} = \kappa_n$, thus using the least possible power, while still ensuring transmission with an arbitrarily low error rate. ■

Compared to the continuous-power transmission scheme (4), discrete-level power control (11) will be suboptimal. As seen from the proof of Lemma 2, this is due to fact that (11) is only optimal at K points ($\gamma_{T_{n,1}}, \dots, \gamma_{T_{n,K}}$) within each pre-adaptation SNR region n , at all other points the transmitted power is greater than what is required for reliable transmission at $\log_2(1 + \kappa_n)$ bits/s/Hz. Clearly, increasing the number of power levels per code K gives a better approximation to the continuous power control (4), resulting in a higher average spectral efficiency. However, as we will see from the numerical results in Section IV, using only a few power levels per code will yield spectral efficiencies close to the upper bound of continuous power adaptation.

Using (11) in (2), (3), we arrive at the following optimization problem:

$$\text{maximize} \quad \sum_{n=1}^N \log_2(1 + \kappa_n) w_n \quad (12a)$$

$$\text{s.t.} \quad \sum_{n=1}^N \kappa_n d_n = 1, \quad (12b)$$

where we have introduced $d_n = \sum_{k=1}^K \frac{1}{\gamma_{T_{n,k}}} \int_{\gamma_{T_{n,k}}}^{\gamma_{T_{n,k+1}}} f_\gamma(\gamma) d\gamma$. As in the case of continuous-power transmission, for fixed $\{\gamma_{T_{n,k}}\}$, (12) is a standard convex optimization problem in $\{\kappa_n\}$, yielding optimal values according to waterfilling as

$$\kappa_n = \frac{w_n}{\lambda d_n} - 1, \quad n = 1, \dots, N, \quad (13)$$

where again λ is a Lagrange multiplier for the power constraint, and from (12b) expressed as

$$\lambda = \frac{1 - F_\gamma(\gamma_{T_{1,1}})}{1 + \sum_{n=1}^N d_n} \quad (14)$$

Then, the optimal switching thresholds $\{\gamma_{T_{n,k}}\}_{n=1, k=1}^{N,K}$ are found by numerically solving the following equation

$$\nabla MASA_{N \times K} = \nabla \sum_{n=1}^N \log_2\left(\frac{w_n}{\lambda d_n}\right) = \mathbf{0}, \quad (15)$$

again simplifying the original constrained optimization problem to solving a set of equations.

C. Constant-Power Transmission Scheme

When a single transmission power is used for all codes, we adopt the term *constant-power transmission scheme*³ [13]. The optimal constant power policy is then to save power when $\gamma < \gamma_{T_{1,1}}$, i.e., when there is no transmission, while transmitting at a power level satisfying (3) with an equality for $\gamma \geq \gamma_{T_{1,1}}$, i.e.,

$$\frac{S(\gamma)}{S} = \begin{cases} \frac{1}{1 - F_\gamma(\gamma_{T_{1,1}})}, & \text{if } \gamma_{T_{n,1}} \leq \gamma < \gamma_{T_{n+1,1}}, 1 \leq n \leq N, \\ 0, & \text{if } \gamma < \gamma_{T_{1,1}}. \end{cases} \quad (16)$$

From (16) we see that the post-adaptation SNR monotonically increases within $[\gamma_{T_{n,1}}, \gamma_{T_{n+1,1}})$, for $1 \leq n \leq N$. Hence, $\log_2(1 + \frac{S(\gamma_{T_{n,1}})}{S} \gamma_{T_{n,1}})$ is the highest possible spectral efficiency that can be supported over the whole of region n . Introducing (16) in (2) we obtain a new expression for the MASA, denoted by $MASA_N$:

$$MASA_N = \sum_{n=1}^N \log_2\left(1 + \frac{\gamma_{T_{n,1}}}{1 - F_\gamma(\gamma_{T_{1,1}})}\right) \int_{\gamma_{T_{n,1}}}^{\gamma_{T_{n+1,1}}} f_\gamma(\gamma) d\gamma. \quad (17)$$

In order to find the optimal set of switching levels $\{\gamma_{T_{n,1}}\}_{n=1}^N$, we first calculate the gradient of the $MASA_N$ —as defined by (17)—with respect to the switching levels. The gradient is then set to zero, and we attempt to solve the resulting set of equations with respect to $\{\gamma_{T_{n,1}}\}_{n=1}^N$:

$$\nabla MASA_N = \begin{bmatrix} \frac{\partial MASA_N}{\partial \gamma_{T_{1,1}}} \\ \vdots \\ \frac{\partial MASA_N}{\partial \gamma_{T_{N,1}}} \end{bmatrix} = \mathbf{0}. \quad (18)$$

³Also termed *on-off* power transmission, see e.g., [6].

For $n = 2, \dots, N$ the partial derivatives in (18) can be expressed as follows:

$$\frac{\partial \text{MASA}_N}{\partial \gamma_{T_{n,1}}} = \log_2(e) \times \left(\frac{\int_{\gamma_{T_{n,1}}}^{\gamma_{T_{n+1,1}}} f_\gamma(\gamma) d\gamma}{1 - F_\gamma(\gamma_{T_{1,1}}) + \gamma_{T_{n,1}}} - \ln \left(\frac{1 - F_\gamma(\gamma_{T_{1,1}}) + \gamma_{T_{n,1}}}{1 - F_\gamma(\gamma_{T_{1,1}}) + \gamma_{T_{n-1,1}}} \right) f_\gamma(\gamma_{T_{n,1}}) \right), \quad (19)$$

where $\ln(\cdot)$ is the natural logarithm. The integral in (19) is recognized as the difference between the cdf of γ , $F_\gamma(\cdot)$, evaluated at the two points $\gamma_{T_{n+1,1}}$ and $\gamma_{T_{n,1}}$. Setting $\frac{\partial \text{MASA}_N}{\partial \gamma_{T_{n,1}}}$ for $2 \leq n \leq N$ equal to zero then yields a set of $N-1$ equations, each with a similar form to the one shown here:

$$F_\gamma(\gamma_{T_{n+1,1}}) - F_\gamma(\gamma_{T_{n,1}}) - (1 - F_\gamma(\gamma_{T_{1,1}}) + \gamma_{T_{n,1}}) \times \ln \left(\frac{1 - F_\gamma(\gamma_{T_{1,1}}) + \gamma_{T_{n,1}}}{1 - F_\gamma(\gamma_{T_{1,1}}) + \gamma_{T_{n-1,1}}} \right) f_\gamma(\gamma_{T_{n,1}}) = 0. \quad (20)$$

Noting that $\gamma_{T_{n+1,1}}$ appears only in one place in this equation, it is trivial to rearrange the $N-2$ first equations into a recursive set of equations where $\gamma_{T_{n+1,1}}$ is written as a function of $\gamma_{T_{n,1}}$, $\gamma_{T_{n-1,1}}$, and $\gamma_{T_{1,1}}$ for $n = 2, \dots, N-1$:

$$\gamma_{T_{n+1,1}} = F_\gamma^{-1} \left[F_\gamma(\gamma_{T_{n,1}}) + (1 - F_\gamma(\gamma_{T_{1,1}}) + \gamma_{T_{n,1}}) \times \ln \left(\frac{1 - F_\gamma(\gamma_{T_{1,1}}) + \gamma_{T_{n,1}}}{1 - F_\gamma(\gamma_{T_{1,1}}) + \gamma_{T_{n-1,1}}} \right) f_\gamma(\gamma_{T_{n,1}}) \right] \quad (21)$$

where $F_\gamma^{-1}[\cdot]$ is the inverse cdf.

For $N \geq 3$, (21) can be expanded in order to yield a set $\gamma_{T_{3,1}}, \dots, \gamma_{T_{N,1}}$ which is optimal for given $\gamma_{T_{1,1}}$ and $\gamma_{T_{2,1}}$. The MASA can then be expressed as a function of $\gamma_{T_{1,1}}$ and $\gamma_{T_{2,1}}$ only. We have now used $N-2$ equations from the set in (18), and the two remaining equations could be used in order to reduce the problem to one equation of one unknown. However, both because of the recursion and the complicated expression for $\frac{\partial \text{MASA}_N}{\partial \gamma_{T_{1,1}}}$, the resulting equation would become prohibitively involved. The final optimization is done by numerical maximization of $\text{MASA}_N(\gamma_{T_{1,1}}, \gamma_{T_{2,1}})$, thus reducing the N -dimensional optimization problem to 2 dimensions. After solving the reduced problem $\gamma_{T_{3,1}}, \dots, \gamma_{T_{N,1}}$ are found via (21).

IV. NUMERICAL RESULTS

One important outcome of the research presented here is the opportunity the results provide for assessing the relative significance of the number of codes and power levels used. It is in many ways desirable to use as few codes and power levels as possible in link adaptation schemes, as this may help overcome several problems, e.g., relating to implementation complexity, adaptation with faulty channel state information (CSI). Thus, if we can come close to the maximum MASA (i.e., the channel capacity) with small values of N and K by choosing our link adaptation schemes optimally, this is potentially of great practical interest.

The constant and discrete schemes offer several advantages considering implementation [29]. In these schemes the transmitter adapts its power and rate from a limited set of values, thus the receiver only need to feed back an indexed rate and

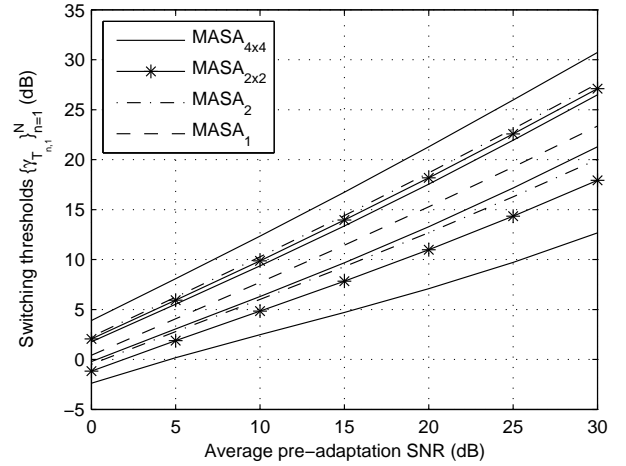


Fig. 3. Switching thresholds $\{\gamma_{T_{n,1}}\}_{n=1}^N$ as a function of average pre-adaptation SNR. For each data series, the lowermost curve shows $\gamma_{T_{1,1}}$, while the uppermost shows $\gamma_{T_{N,1}}$.

power pair for each fading block. Obviously, compared to the feedback of continuous channel state information, this results in reduced requirements of the feedback channel bandwidth and transmitter design. Further, completely discrete schemes are more resilient towards errors in channel estimation and prediction.

Two performance merits will be taken into account: The MASA, representing an approachable upper bound on the throughput when the scheme is under the restriction of a certain number of codes and power adaptation flexibility, and the probability of no transmission, $P_{\text{no tr.}}$.

A. Switching Levels and Power Adaptation Schemes

Fig. 3 shows the set of optimal switching levels $\{\gamma_{T_{n,1}}\}_{n=1}^N$ for selected MASA schemes and for $0 \text{ dB} < \bar{\gamma} < 30 \text{ dB}$. (For the $\text{MASA}_{2 \times 2}$ and $\text{MASA}_{4 \times 4}$ schemes the internal switching thresholds $\{\gamma_{T_{n,k}}\}_{n=1, k=2}^{N,K}$ are not shown in Fig. 3 due to clarity reasons). Table I shows numerical values, correct to the first decimal place, for designing optimal systems with $N = 4$ at $\bar{\gamma} = 10 \text{ dB}$. Fig. 3 and Table I should be interpreted as follows: With the mean pre-adaptation SNR $\bar{\gamma}$, the number of codes N and a power adaptation scheme in mind, find the set of switching levels and the corresponding maximal spectral efficiencies, given by

$$\text{SE}_n = \begin{cases} \log_2 \left(1 + \frac{\gamma_{T_{n,1}}}{1 - F(\gamma_{T_{1,1}})} \right) & \text{for MASA}_N, \\ \log_2(1 + \kappa_n) & \text{for MASA}_{N \times K} \text{ and MASA}_{N \times \infty}. \end{cases} \quad (22)$$

Then design optimal codes for these spectral efficiencies, for each $\bar{\gamma}$ of interest.

Examples of optimized power adaptation schemes are shown in Fig. 4, illustrating the piecewise channel inversion power adaptation schemes of the $\text{MASA}_{N \times K}$ and $\text{MASA}_{N \times \infty}$ schemes. For $\gamma \leq 15 \text{ dB}$ the discrete-power scheme of $\text{MASA}_{4 \times 4}$ closely follows the continuous power adaptation scheme of $\text{MASA}_{4 \times \infty}$. Fig. 4 also depicts the optimal power allocation (denoted C_{OPRA}) for continuous-rate adaptation [20,

TABLE I
RATE AND POWER ADAPTATION FOR FOUR REGIONS, $\bar{\gamma} = 10$ dB

	MASA ₄	MASA _{4×4}	MASA _{4×∞}
$\gamma_{T_{1,1}}, \dots, \gamma_{T_{4,1}}$ (dB)	4.4, 7.3, 9.8, 12.4	2.5, 6.3, 9.4, 12.3	1.4, 5.5, 8.9, 12.3
$\kappa_1, \dots, \kappa_4$	-	2.4, 6.6, 13.9, 29.0	2.0, 6.0, 13.8, 31.3
SE_1, \dots, SE_4	1.9, 2.7, 3.4, 4.2	1.8, 2.9, 3.9, 4.9	1.6, 2.8, 3.9, 5.0

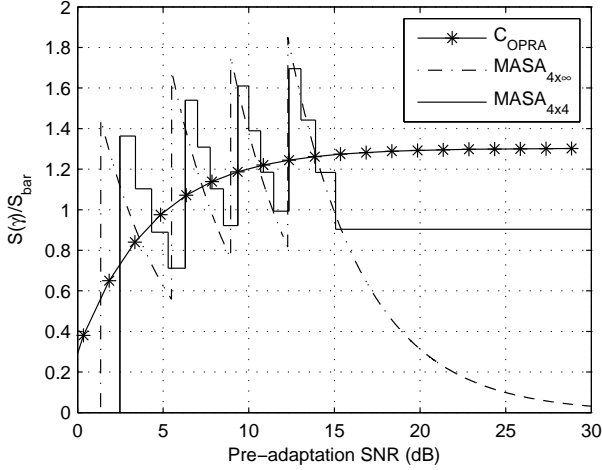


Fig. 4. Power adaptation schemes for MASA_{4×∞} and MASA_{4×4} as a function of pre-adaptation SNR, plotted for an average pre-adaptation SNR $\bar{\gamma} = 10$ dB. Optimal power adaptation for continuous-rate adaptation C_{OPRA} as reference.

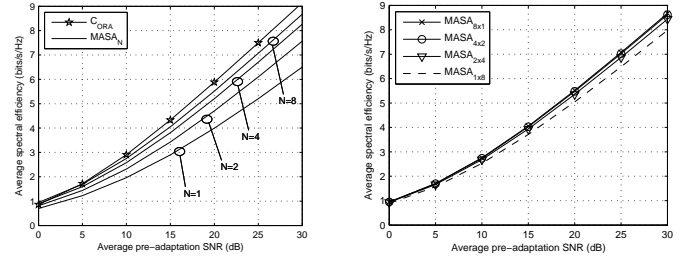
Eq. 5]. At $\bar{\gamma} = 10$ dB, two discrete-rate MASA schemes allocate more power to codes with higher spectral efficiency, following the water-filling nature of C_{OPRA} . In the analysis of Section III no stringent peak power constraint has been imposed, and it is interesting to note the limited range of $S(\gamma)$ that still occurs for both MASA_{4×4} and MASA_{4×∞}.

B. Comparison of MASA Schemes

Under the average power constraint of (3) the average spectral efficiencies corresponding to MASA_N, MASA_{N×K}, and MASA_{N×∞} are plotted in Figs. 5 and 6. From Fig. 5(a) we see that the average spectral efficiency increases with the number of codes, while Fig. 6 shows that the ASE also increases with flexibility of power adaptation.

Fig. 5(b) compares four MASA schemes with the product $N \times K = 8$, showing that number of codes has a slightly larger impact on the spectral efficiency than the number of power levels. However, we see that the three schemes with $N \geq 2$ have almost similar performance, indicating that the number of rates and power levels can be traded against each other, while still achieving approximately the same ASE. From an implementation point of view this is valuable as it gives more freedom to design the system.

Finally, as mentioned in the introduction, there are at least two distinct design philosophies for link adaptation systems, depending on whether the number of *regions* in the partition of the pre-adaptation range γ or the number of *rates* is the starting point of the design, and correspondingly on whether



(a) Average spectral efficiency of (b) Average spectral efficiency of MASA_N for $N = 1, 2, 4, 8$ and MASA_{N×K} as a function of $\bar{\gamma}$, for C_{ORA} for reference. four MASA schemes with $N \times K = 8$.

Fig. 5. Average spectral efficiency of different MASA schemes.

information *outage* can be tolerated. Now, a direct comparison is not possible, but to highlight the differences between the two philosophies we provide a numerical example.

Example 1: Consider designing a simple rate-adaptive system with two regions, where the goal is to maximize the expected rate using a single power level per region. Assuming the average pre-adaptation SNR on the channel to be 5 dB and following the setup of [16]–[18], we find the maximum *average reliable throughput* (ART), defined as the “average data rate assuming zero rate when the channel is in outage” [16] that can be achieved to be 1.2444 bits/s/Hz, and that the probability of information outage, or equivalent the probability that an arbitrary transmission will be corrupted, is 0.3098. Thus, without retransmissions, the system is likely to be useless for many applications.

Now, turning to the MASA schemes discussed in this paper, using two regions, but only one constellation and power level, i.e., MASA₁, we see from Fig. 5(a) that this scheme achieves a spectral efficiency of 1.2263 bits/s/Hz at $\bar{\gamma} = 5$ dB without outage. This is only marginally less than the scheme from [16]–[18] when using two constellations and allowing for a non-zero outage.

C. Comparison of MASA schemes with Shannon Capacities

Assume that the channel state information γ is known to the transmitter and the receiver. Then, given an average transmit power constraint the channel capacity of a Rayleigh fading channel with optimal *continuous* rate adaptation and constant transmit power, C_{ORA} , is given in [20], [30] as

$$C_{ORA} = \log_2(e) e^{\frac{1}{\bar{\gamma}}} E_1\left(\frac{1}{\bar{\gamma}}\right), \quad (23)$$

where $E_1(\cdot)$ is the exponential integral of first order [31, p. xxxv]. Furthermore, if we include *continuous* power adap-

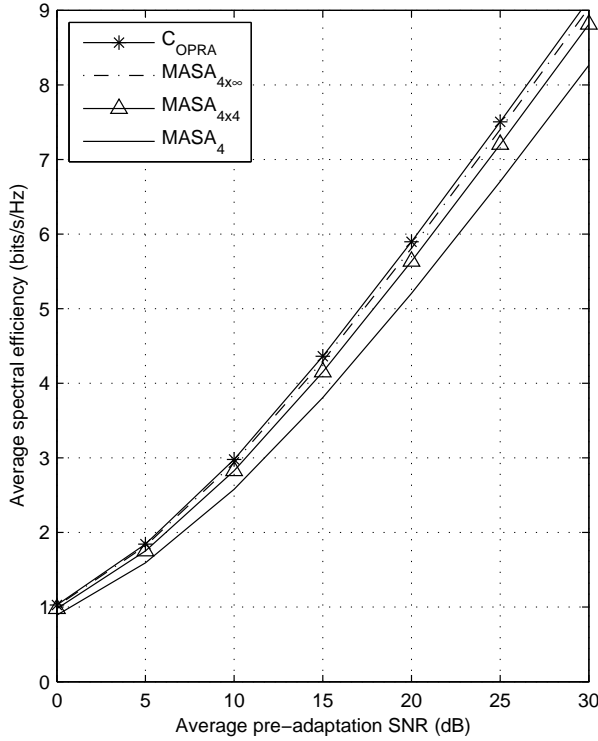


Fig. 6. Average spectral efficiency for various MASA schemes with $N = 4$ codes as a function of $\bar{\gamma}$. C_{OPRA} as reference.

tation, the channel capacity, C_{OPRA} , becomes [20], [30]

$$C_{OPRA} = \log_2(e) \left(e^{\frac{-\gamma_{cut}}{\bar{\gamma}}} - \bar{\gamma} \right), \quad (24)$$

where the “cutoff” value γ_{cut} can be found by solving

$$\int_{\gamma_{cut}}^{\infty} \left(\frac{1}{\gamma_{cut}} - \frac{1}{\gamma} \right) f_{\gamma}(\gamma) d\gamma = 1. \quad (25)$$

Thus, $MASA_N$ is compared to C_{ORA} , while $MASA_{N \times K}$ and $MASA_{N \times \infty}$ are measured against C_{OPRA} .

The capacity in (24) can be achieved in the case that a continuum of capacity-achieving codes for AWGN channels, and corresponding optimal power levels, are available. That is, for each SNR there exists an optimal code and power level. Alternatively, if the fading is ergodic within each codeword, as opposed to the assumptions in this paper, C_{OPRA} can be obtained by a fixed rate transmission system using a single Gaussian code [22], [32].

As the number of codes (switching thresholds) goes to infinity, $MASA_N$ will reach the C_{ORA} capacity, while $MASA_{N \times K}$ will reach the C_{OPRA} capacity when $N, K \rightarrow \infty$. Of course this is not a practically feasible approach; however, as illustrated in Figs. 5(a) and 6, a small number of optimally designed codes, and possibly power adaptation levels, will indeed yield a performance that is close to the theoretical upper bounds, C_{ORA} and C_{OPRA} , for any given $\bar{\gamma}$.

From Fig. 6 we see that the power adapted MASA schemes perform close to the theoretical upper bound (C_{OPRA}) using only four codes. Specifically, restricting our adaptive policy to just four rates and four power levels per rate results in

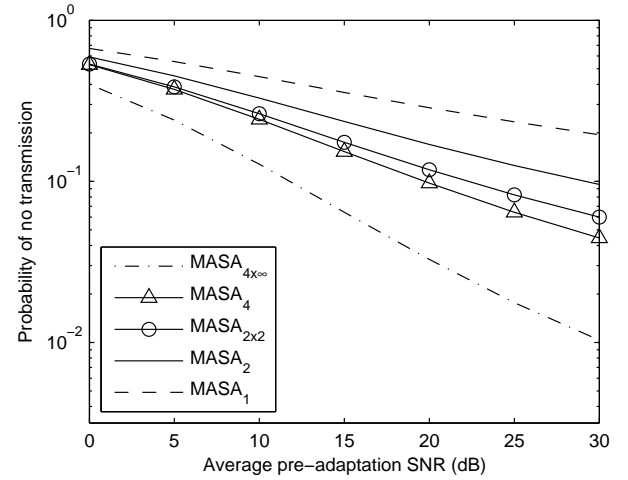


Fig. 7. The probability of no transmission $P_{no \text{ tr.}}$ as a function of average pre-adaptation SNR.

a spectral efficiency that is within 1 dB of the efficiency obtained with continuous-rate and continuous-power (24), demonstrating the remarkable impact of power adaptation. This is in contrast to the case of continuous rate adaptation, where introducing power adaptation gives negligible gain [20].

D. Probability of no transmission

When the pre-adaptation SNR falls below $\gamma_{T_{1,1}}$ no data are sent. The probability of no transmission $P_{no \text{ tr.}}$ for the Rayleigh fading case can then be calculated as follows,

$$P_{no \text{ tr.}} = \int_0^{\gamma_{T_{1,1}}} f_{\gamma}(\gamma) d\gamma = 1 - e^{-\frac{\gamma_{T_{1,1}}}{\bar{\gamma}}}. \quad (26)$$

When the number of codes is increased, the SNR range will be partitioned into a larger number of regions. As shown in Fig. 3, the lowest switching level $\gamma_{T_{1,1}}$ will then become smaller. $P_{no \text{ tr.}}$ will therefore decrease, as illustrated in Fig. 7. Similarly, as seen from Fig. 3, $\gamma_{T_{1,1}}$ also decreases with an increasing number of power levels, when N is constant. Thus, both rate and power adaptation flexibility reduces the probability of no transmission.

For applications with low delay requirements, it could be beneficial to enforce a constraint that $P_{no \text{ tr.}}$ should not exceed a prescribed maximal value. Then, we may simply—using (26)—compute $\gamma_{T_{1,1}}$ to be the highest SNR value which ensures that this constraint is fulfilled. The MASA schemes are then optimized to obtain the highest possible ASE under the given constraint on no transmission, i.e., optimization with $\gamma_{T_{1,1}}$ as a pre-determined parameter. As an example, in Fig. 8, the obtainable average spectral efficiency for the $MASA_N$ scheme with the additional constraint that $P_{no \text{ tr.}} \leq 10^{-3}$ (dashed lines) is compared to the case without a constraint on no transmission probability (solid lines). We see that for $N = 2$ the constraint has a severe influence on the ASE, while for $N = 8$ the constraint can be fulfilled without significant losses in spectral efficiency.

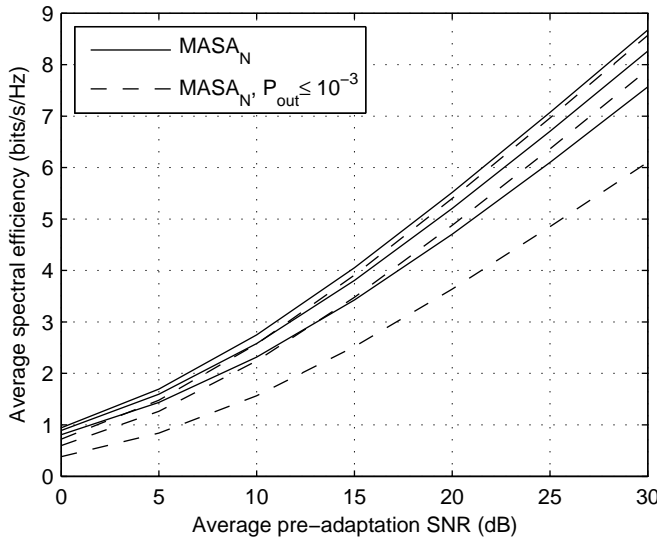


Fig. 8. MASA_N as a function of $\bar{\gamma}$, with a constraint on the probability of no transmission (solid lines) and without (dashed lines). Plotted for $N = 2$ (lowermost curve for both series), 4, and 8 (uppermost curve for both series).

V. CONCLUSIONS AND DISCUSSIONS

Using a zero information outage approach, and assuming that capacity-achieving component codes are available, we have devised spectral efficiency maximizing link adaptation schemes for flat block-fading wireless communication channels. Constant, discrete, and continuous-power adaptation schemes are proposed and analyzed. Switching levels and power adaptation policies are optimized in order to maximize the average spectral efficiency for a given fading distribution.

We have shown that a performance close to the Shannon limits can be achieved with all schemes using only a small number of codes. However, utilizing power adaptation is shown to give significant average spectral efficiency and probability of no transmission gains over the constant transmission power scheme. In particular, using a fully discrete scheme with just four codes, each associated with four power levels, we achieve a spectral efficiency within 1 dB of the Shannon capacity for continuous rate and power adaptation. Additionally, constant and discrete-power adaptation schemes render the system more robust against imperfect channel estimation and prediction, reduce the feedback load and resolve implementation issues, compared to continuous power adaptation.

We have also seen that the number of rates N can be traded against the number of power levels K . This flexibility is of practical importance since it may be easier to implement the proposed power adaptation schemes than to design capacity-achieving codes for a large number of rates. The analysis can be augmented to encompass more practical scenarios, e.g., by taking imperfect CSI [33] and SNR margins due to various implementation losses, into account. Finally, we note that the adaptive power algorithms presented in this paper require that the radio frequency (RF) power amplifier is operated in the linear region, implying a higher power consumption. For devices with limited battery capacity it is apparent that there

will be a tradeoff between efficiency and linearity. This can be a topic for further research.

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